

## On fully-dispersed shock waves in carbon dioxide

By WAYLAND C. GRIFFITH and ANNE KENNY

*Palmer Physical Laboratory, Princeton, New Jersey*

(Received 26 June 1957)

A dispersed shock wave may be defined as one in which finite changes occur over distances large compared to the mean free path in the gas. In contrast a shock wave in air extends over only a few mean free paths. When internal motions in a molecule are excited rather slowly by collisions, as is the case for molecular vibration, the shock wave may be partly dispersed; then, the sharp shock front is followed by a diffuse tail leading to complete thermal equilibrium. Alternatively, it may be fully dispersed, so that adjustments in the energy in all the degrees of freedom proceed slowly and in parallel. The purpose of this note is to point out that within a narrow speed range, from a shock Mach number of 1 to 1.042, shocks in carbon dioxide are fully-dispersed in the above sense. Such waves have been observed experimentally using a shock tube and interferometer. The possible existence of such waves was first pointed out by Bethe & Teller (1941) purely as a matter of academic interest. This note treats the problem in the same spirit.

Lighthill (1956) views the problem of shock structure as one of finding a balance between convection of wave energy forward through the gas passed over by the shock and diffusion of energy by conduction, viscous friction, and vibrational relaxation. The latter process, described by the relaxation equation

$$\frac{dE_{\text{vib}}}{dt} = \frac{E'_{\text{vib}} - E_{\text{vib}}}{\tau}, \quad (1)$$

dominates the other two in the phenomenon considered here.  $E_{\text{vib}}$  is the energy actually in the vibrational modes, the primed quantity is the energy appropriate to equilibrium, and  $\tau$  is the vibrational relaxation time.

The speed of sound in an ideal gas is given by

$$a = (\gamma RT)^{1/2}. \quad (2)$$

Because the vibrational heat capacity cannot respond to sudden changes in the state of the gas, however, we see that a sharp shock front, suitable as the head of a partly-dispersed shock, can form only when the wave speed is greater than the speed of sound computed by not counting in the vibrational contribution to heat capacity. For any linear molecule this is

$$a' = (1.4RT)^{1/2}. \quad (3)$$

In the speed range from  $M = 1$  to  $M = (1.4/\gamma)^{1/2}$  a fully-dispersed wave must form. Its shape will be governed by equation (1).

Lighthill (1956) gives an approximate solution for the velocity profile of such a fully-dispersed shock wave, which is valid for very weak waves. More exact analysis gives

$$\frac{X}{v_1 \tau} = \frac{(\gamma - 1)}{(\gamma' - 1)(M_1^2 - 1)} \left[ \left( \frac{a_1'^2}{a_1^2} - M_1^2 \right) \log \left( 1 - \frac{v}{v_1} \right) - \frac{a_2'^2}{a_1^2} \left( 1 - \frac{v_2^2}{a_2'^2} \right) \log \left( \frac{v - v_2}{v_1} \right) \right]. \quad (4)$$

Here, subscripts 1 and 2 refer to equilibrium conditions ahead of and behind the shock, respectively, and the primed quantities are to be computed without the inclusion of  $E_{vib}$ . For pure carbon dioxide at 300° K and 1 atmosphere,  $\tau = 10 \mu\text{sec}$ ,  $\gamma = 1.290$ , and  $a = 880 \text{ ft./sec}$ . A shock front would therefore be expected for  $M \geq 1.042$ . Profiles for  $M = 1.01, 1.02, 1.03$  and  $1.04$  are shown in figure 1. Below  $M = 1.01$  the wave is symmetrical and of the same spatial extent as that shown for this Mach number.

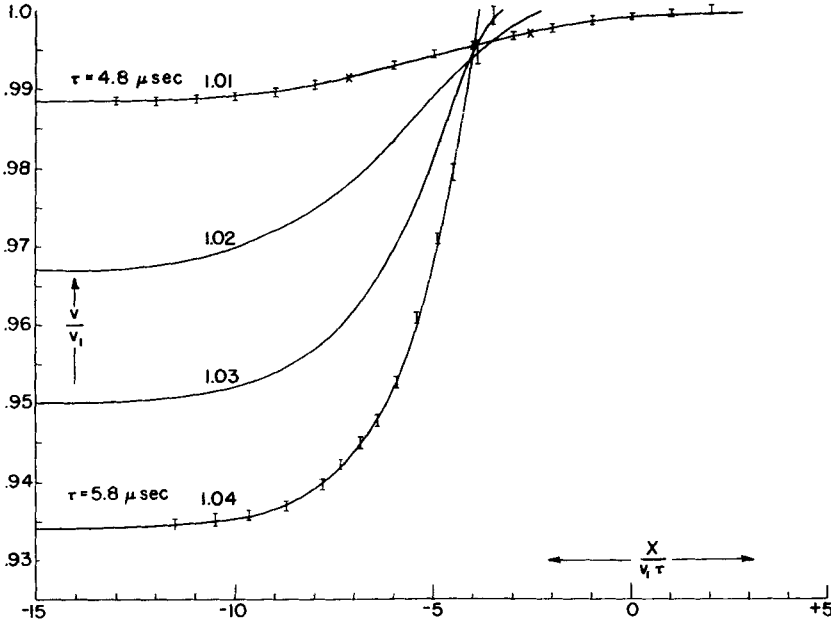


Figure 1. Calculated velocity profiles for shocks of Mach number 1.01, 1.02, 1.03 and 1.04 in carbon dioxide. The experimental data were obtained from the interferograms shown in figures 2 and 3 (plate 1). The one-quarter and three-quarters points are indicated by a cross.

The experimental values plotted on figure 1 were obtained from such interferograms as those shown in figures 2 and 3 (plate 1). Shock strengths were computed from the total observed fringe shifts since these waves are too weak to trip any of our timing circuits. The effective relaxation

time for each experiment was obtained by fitting the points corresponding to one-quarter and three-quarters of the total fringe shifts to the appropriate theoretical curve. This was necessary because the relaxation time, which determines the spatial scale factor, is very sensitive to minute gas impurities which we cannot control with precision. The values of  $\tau$  thus computed are attached to the curves. It may be seen that these are somewhat below what we believe to be the value for a pure gas.

All gases with a mode that takes up energy over many collisions may be expected to exhibit a similar effect. The reason why it has not been seen before lies in the small vibrational heat capacity of most common gases, which makes the velocity shift small, or the very short relaxation time of most polyatomic gases, which makes the width of the diffuse wave small. Carbon dioxide offers the most favourable combination of these two properties. Above  $M = 1.042$  a shock front is observed and is followed by a relaxation region, as reported by Griffith, Brickl & Blackman (1956).

This work was supported by the Office of Naval Research. We are indebted to M. J. Lighthill for a stimulating discussion which led to this work.

#### REFERENCES

- BETHE, H. & TELLER, E. 1941 *Deviations from thermal equilibrium in shock waves*, Cornell University unpublished report.
- GRIFFITHS, W., BRICKL, D. & BLACKMAN, V. 1956 *Phys. Rev.* **102**, 1209.
- LIGHTHILL, M. J. 1956 *Viscosity in waves of finite amplitude*; article in *Surveys in Mechanics*. Cambridge University Press.

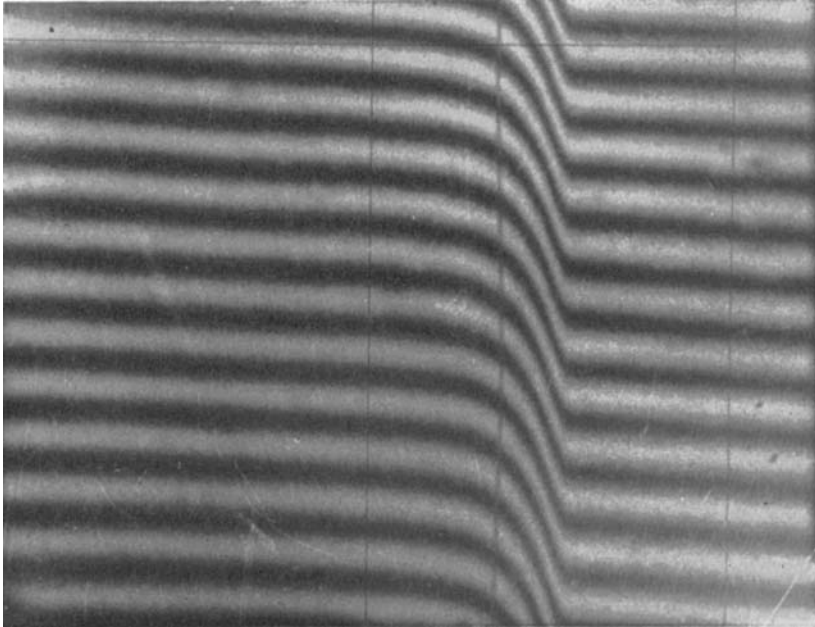


Figure 2. Interferogram of diffuse wave travelling to the right through carbon dioxide at a Mach number of 1.04. Vertical displacement of the fringes is proportional to the density. The outer two hairlines are one inch apart.

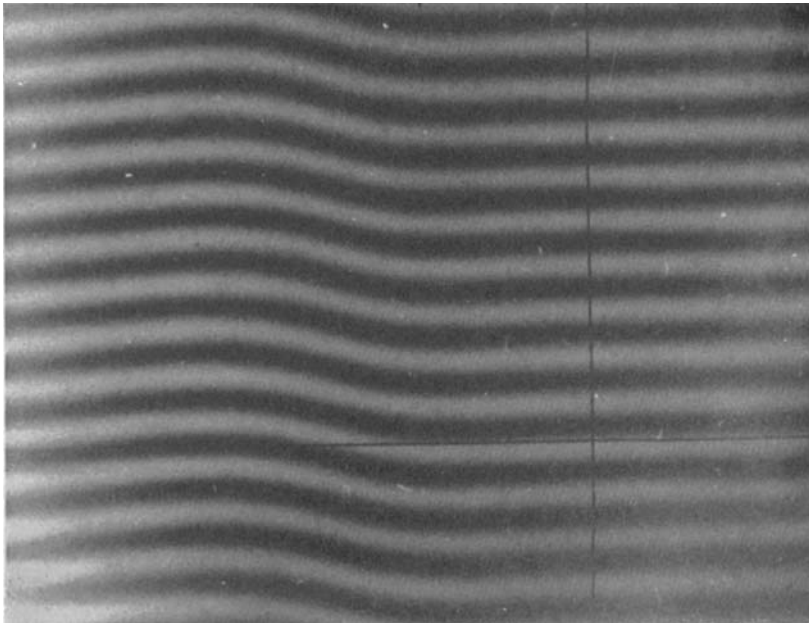


Figure 3. A wave of Mach number 1.01.